LA TRANSFORMADA Z EN SISTEMAS DE CONTROL: REVISIÓN Y UN CASO DE APLICACIÓN

THE Z-TRANSFORM INTO CONTROL SYSTEMS: REVIEW AND A CASE OF APPLICATION

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Resumen

El presente documento describe el resultado de investigación de carácter exploratorio que condujo a estructurar una revisión categorizando y subcategorizando la transformada Z y sus aplicaciones en los sistemas de control. Para lo anterior, se usa el método por índices o basado en una taxonomía categórica orientada por fuentes clasificadas con las claves: historia, conceptualización, aplicación en señales elementales, criterios y características constructivas, y análisis y diseño en sistemas digitales; la metodología se valida por el juicio de expertos del grupo de investigación SciBas. Finalmente se desarrolla una aplicación en MatLab® simulando una implementación de un sistema de control. Como consecuencia se establece una línea de base de conocimiento para investigaciones sobre aplicaciones basadas en la transformada Z, y una forma de abordar simulaciones como alternativa de validación de resultados en un caso particular de sistemas de control.

Palabras clave: transformada Z, caracterización, aplicaciones, sistemas de control, instrumentación.

Abstract

This document describes the result of an exploratory research that led to structuring a review categorizing and subcategorizing the Z transform and its applications in control systems. For the above, the method by indexes or based on a categorical taxonomy oriented by sources classified with the keys is used: history, conceptualization, application in elementary signals, criteria and constructive characteristics, and analysis and design in digital systems; the methodology is validated by the judgment of experts from the SciBas research group. Finally, an application is developed in MatLab® simulating an implementation of a control system. As a consequence, a knowledge base line is established for research on applications based on the Z transform, and a way of approaching simulations as an alternative to validate results in a particular case of control systems.

Keywords: Z-transform, characterization, applications, control systems, instrumentation.

Recibido: 10 de septiembre de 2020 /Evaluación: 25 de octubre de 2020 / Aprobado: 12 de diciembre de 2020

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From the beginning, two problems have been important in probability theory: obtaining the sum probability distribution of *n* random variables –each one of them obeying a discrete probability law with *k* equidistant variable values–; and the development of asymptotic forms of such distributions, valid for *n* increments. Thus emerged the Probability Generating Functions (PGF) that simplified both situations.

In the above scenario, it is equivalent to consider throwing a generalized dice with k faces, any one of them equally probably to appear when the dice is thrown, and ask what is the probability that such *n* dice will yield a total of *x* points – k^n –. Abraham de Moivre, around 1730, gave the problem its first algebraic solution (Seal, 1949), expressed in terms of the favorable *chances* (combinations) and total possible *chances*, indicating that the total *chances* on any of the dice can be represented algebraically by (1):

$$\sum_{n=1}^{k} t^n = t + t^2 + t^3 + \cdots + t^k \qquad (1)$$

And since the power of any *t* denotes the number of points on its corresponding face, it generalizes the result for all *chances* of two, three and any *n* of such dice by raising respectively (1) squared, cubed, or to the *n*. Thus it devised a PGF, (2):

$$\varphi(z) = \sum_{x=1}^{k} \frac{1}{k} z^x = \frac{1}{k}[z + z^2 + z^3 + \cdots + z^k]$$
$$= \frac{1}{k}\left[\frac{z - z^{k+1}}{1 - z}\right] \qquad (2)$$

So the probability that the PGF of the sum of *n* discrete variables is indicated by (3):

$$\varphi_n(z) \approx \sum_{x=n}^{kn} P_n(x) z^x = [\varphi(z)]^n = \frac{z^n}{k^n}\left[\frac{z - z^{k+1}}{1 - z}\right] = \frac{z^n}{k^n}[1 - z^k]^n[1 - z]^{-n} \qquad (3)$$

$P_n(x)$ is the required probability of a total of *x* points with *n* dice, and after a simple count with binomial expansions reduces it to (4):

$$P_n(x) = \frac{1}{k^n}\sum_{j=0}^{m}(-1)^j \binom{n}{j}\binom{x - kj - 1}{x - n - kj} \qquad (4)$$

Where $m = mín.(n, \left(\frac{x-n}{k}\right))$, (Seal, 1949).

De Moivre's proposal was followed and generalized by his contemporaries –Simpson, Lagrange, among others– when they calculated the probability of a total error *x* which is expressed as the sum of *w* individual errors; only that $\varphi(z)$ appears with a factor z^{-h} with a discontinuity to be removed at the origin or center of the distribution (5) and (6):

$$\varphi(z) = \sum_{x=-h}^{h} P(x) z^x = \frac{z^{-h}}{(h+1)^2}[1 - z^{k+1}]^2[1 - z]^{-2} \qquad (5)$$

And, raising to the *n*: $\varphi_n(z) = \frac{z^{-nh}}{(h+1)^{2n}}[1 - z^{k+1}]^{2n}[1 - z]^{-2n}$ (6)

If reasoning arbitrarily in (6) taking $\lim_{h\to\infty}\left(\varphi_n(z)\right)$, overusing the notation it can be written in a general way as (7):

$$\sum_{n=-\infty}^{\infty} P_n(z) z^{-n} \qquad (7)$$

Which surprisingly takes an analogous form (8), noted two centuries later as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \qquad (8)$$

In (8) $x(n)$ is assumed to be the sample of a signal, or sequence, and is considered transformed by multiplying by z^{-k}, and then summing.

This formulation dates back to Laplace, but was formally introduced by Witold Hurewicz in 1947 to solve linear equations in differences with constant coefficients or an equation recurrently defined by a sequence that serves as an unknown to be determined (Jury, 1964). In the case of a first order linear ecuation, *k=1, m≥0*; $c(n), x(n)$ sequences with known $x(n)$, are expressed in (9):

$$\sum_{n=0}^{k} c(n)z_{m+n} = x(n) \qquad (9)$$

Again, Abraham de Moivre solves it in *Miscellanea analytica de seriebus et quadraturis* (De Moivre, 1730). However, the first text that dealt exhaustively with equations in differences is written towards the end of the XIX century with the last edition of *Finite Differences* by George Boole.

However, it was not until 1952 that (8) was called the Z-transform – probably because of the many Z's in the surnames of J. R. Raggazini and L. A. Zadeh, who composed the *sampled-data control* research group, which also included E. I. Jury, R. E. Kalman, J. E. Bertram, B. Friedland and G. F. Franklin, at Columbia University. That is: the Z transform should be called the De Moivre or Hurewicz transform.

In practice, the Z transform has been effective in the analysis of discrete phenomena, as well as in signal analysis and control systems in the discrete domain. (James et al., 2002), (Brea, 2014), a linear difference equation is always associated with it, characterizing the system dynamics. In this way, to determine the system response to a given input, this equation in differences must be solved Among these operational methods, the Z transform method is the one that transforms the solutions to difference equations into a problem of an algebraic nature (Ogata, 2003). The Z transform (ZT) the Z transform has the same role in linear time-invariant (LTI) systems as the Laplace transform does in continuous signal and system analysis, and is a generalization of the discrete Fourier transform. (Soria Olivas et al., 2003).

In other words: the Z transform is a mathematical model used in the study of Digital Signal Processing, Digital Circuits, Radar or Telecommunication Systems and especially Computer Process Control Systems (Sacerdoti, 2010). These models have been spread by the increasing development of communication theory and advances in computer systems, which have allowed to bring solutions to the digital field, penetrating almost all activities of the modern engineering society (Cánovas Peña, 2009).

Therefore, this paper provides a descriptive documentary research that shows the categorization and subcategorization of the Z transform and its applications, with the aim of having a baseline to guide research on innovation and development applied to control engineering and instrumentation of digital technological systems. In this sense, a study case applied in the field of control engineering is developed.

Background and notation

(El Attar, 2006; Ward Brown & Churchill, 2009; Sami Fadali & Visioli, 2019; Hayes, 1999; Oppenheim & Willsky, 1996; Proakis & Manolakis, 1990; Shilling, 1970; Fazarinc, 2013).

From no won, $x(n)$ is understood as a complex-valued function $(x\colon \mathbb{N} \rightarrow \mathbb{C})$ or complex sequence or discrete signal; z is a complex variable $(z \in \mathbb{C})$; likewise, given an analytic function $f(z)$ in a ring $r < |z - z_0| < R$,(10); and given a simple closed curve C around z_0 positively oriented and contained in the ring then, at each point of that domain $f(z)$ can be represented by a Laurent Series (10):

$$f(z) = \sum_{n=-\infty}^{\infty} x(n)(z - z_0)^n \quad ,r < |z - z_0| < R \qquad (10)$$

$$x(n) = \frac{1}{2i\pi} \oint \frac{f(z)}{(z - z_0)^{n+1}} dz, \;\; n \in \mathbb{Z} \qquad (11)$$

Also, if a $f(z)$ function s analytic in and on a simple closed positively oriented curve, except at a finite number of singularities z_k, $k = 1,2, \dots, n$ inside C, then there is Cauchy Residue Theorem:

$$\frac{1}{2i\pi} \oint f(z) dz = \sum_{k=1}^{n} Res[f(z), z_k] \qquad (12)$$

Then, the bidirectional Z transform is defined as in (8):

$$X(z) \equiv Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

The region of convergence (ROC) of $X(z)$, is the set of all values of z where $X(z)$ is finite, (15):

$$|X(z)| = \left| \sum_{n=-\infty}^{\infty} x(n) z^{-n} \right| < \infty \qquad (15)$$

Since this transform does not determine a unique signal in the time domain, it is recognized that Z transform is specified by ROC (Soria Olivas et al., 2003).

The relationship between $x(n)$ –inverse transform– y $X(z)$ – direct transform– is given as follows (16):

$x(n) \overset{z}{\leftrightarrow} X(z)$, (16) (James et al., 2002), (Oppenheim & Schafer, 2001).

Inverse transform is defined as (17):

$$x(n) = \frac{1}{2\pi j} \int_C X(z) z^{n-1} dz, \qquad (17)$$

Where C is any simple closed curve containing the origin and lying on *X(z)* ROC in the Z plane; the line integral is meant on a counterclockwise circle.

The basic forms for inverse Z transform calculation are based on direct line integration (17); or by expansion of $X(z)$ in power series; or by expansion of $X(z)$ in simple fractions. Since the sequences that are usually worked with are nearly the same, the last two methods are the preferred ones because of their simplicity. (Soria Olivas et al., 2003).

As previously mentioned, the need for an analogy with the Laplace transform to solve finite difference equations is still valid when modeling electronic systems whose inputs and outputs are a succession of discrete data (Milone et al., 2009). The above is done by discretizing time *t* into the direct Laplace transform $F(s) = \int_0^{+\infty} f(t) e^{-st} dt$, taking integer increments *n* to obtain a direct discrete causal transform:

Table 1. Z transform of elementary functions (Rondón Carreño, 2005)**.**

$$\sum_{n=0}^{+\infty} f(n)e^{-sn} \qquad (18)$$

Using the

ITEM	FUNCTION	*Z* TRANSFORM
1	Step	$X(z) = \frac{z}{z-1}$
2	Ramp	$X(z) = \frac{Tz}{(z-1)^2}$
3	Polynominal	$X(z) = \frac{z}{z-a}$
4	Exponential	$X(z) = \frac{z}{z-e^{-aT}}$
5	Sinusoidal	$X(z) = \frac{\mathrm{z}\sin(wT)}{z^2-2z\cos(wT)+1}$
6	Cosenoidal	$X(z) = \frac{z^2-zcos(wT)}{z^2-2z\cos(wT)+1}$

substitution $z = e^s$, an equivalent function $F(s)$ can be written but in terms of z (19):

$$\sum_{n=0}^{+\infty} f(n)z^{-n} \qquad (19)$$

This is a causal –unidirectional– form of the Z transform. On the other hand, in an inverse Laplace transform, the following is done analogously (20):

$$f(n) = \frac{1}{2\pi j}\int_{0}^{+\infty} F(z)e^{sn}ds \qquad (20)$$

The $\frac{dz}{z}$ differential is found directly by explicitly deriving in the expression$z = e^s$ and substituting (20), obtaining (21):

$$f(n) = \frac{1}{2\pi j}\int_{z=-\infty}^{+\infty} F(z)z^{n-1}dz \qquad (21)$$

Tables 1 and 2 show the Z transform of some elementary functions, as well as the properties that they fulfill.

Table 2. ***Z*** **transform properties** (James et al., 2002), (Ogata, 2003), (Oppenheim & Schafer, 2001), (Rondón Carreño, 2005).

ITEM		DEFINITION

	PROPERTY OR THEOREM	
1	Multiplication by an exponential sequence	Multiplication by $a^k = Z\{a^k x_k\} = X(a^{-1}z)$ Multiplication by $k^n = Z\{k^n x_k\} = \left(-z\frac{d}{dz}\right)^n X(z)$
2	Initial value theorem	$\lim_{z\to\infty} X(z) = x_0$
3	Final value theorem	$\lim_{k\to\infty} x_k = \lim_{z\to 1}(1-z^{-1})X(z)$
4	First translation property (delay)	$Z\{x_{k-k_0}\} = \frac{1}{Z^{k_0}} Z\{x_k\}$
5	Second translation property (advance)	$Z\{x_{k+1}\} = zX(z) - zx_0$
6	Multiplication by a constant	$Z[ax(t)] = \sum_{k=0}^{\infty} ax(kT)z^{-k} = a\sum_{k=0}^{\infty} x(kT)z^{-k} = aX(z)$
7	Linearity	$X(z) = \alpha Z[f(k)] + \beta Z[g(k)]$
8	Time displacement	$x[k-k_0] = Z[z^{-k_0}x(z)]$
9	X(z) Differentiation	$nx[n] \overset{Z}{\leftrightarrow}$ -z $\frac{dX(z)}{dz}$, $ROC = R_X$
10	Temporary reversal	$x[-n] \overset{Z}{\leftrightarrow} X(1/z)$, $ROC = \frac{1}{R_x}$
11	Succession Convolution	$x_1[n] * x_2[n] \overset{Z}{\leftrightarrow} X_1(z)X_2(z),\ ROC\ contains\ R_{x1}\ \ R_{x2}.$
12	Complex translation theorem	$Z[e^{-at}x(t)] = \sum_{k=0}^{\infty} x(kT)e^{-akT}z^{-k} = \sum_{k=0}^{\infty} x(kT)(ze^{aT})^{-k} = X(ze^{at})$

Methodology

To build the review an index method is applied. It is based on a categorical taxonomy of the Z transform and its applications in control systems oriented by classified sources with the following keys: history, conceptualization, application in elementary signals, criteria and constructive characteristics, analysis and design in digital systems; this methodology is validated by expert judgement of the *SciBas* research group (Hernandez Sampieri et al., 2010).

The study sources were obtained and explored in paper indexing and referencing systems and databases: Google Scholar, *IEEE*, and *SCOPUS*.

The categorization and subcategorization of the Z transform is shown in Figures 1 and 2.

Figure 1 presents the first classification segment where the properties and theorems of both the transform and its inverse and the functions to be obtained are established, as well as the analysis methods and design types to be obtained.

Figure 2 shows the second classification segment where is established: industrial applications (focused on Control and Instrumentation); medical; and telecommunications; as well as application areas.

Table 3 lists all 71 sources that were chosen in each application.

Table 3. Selected sources in Z transform implementations in different industrial, medical and telecommunications applications. Source: Own elaboration

INDUSTRIAL	**CONTROL**	(Orjuela & Chica, 2007), (Pérez Ramírez et al., 2019), (Bory-prevez & Chang-Mumañ, 2017), (Kojima et al., 2004), (Sabin & González, 2009), (Moroz & Holovach, 2017), (Pérez R et al., 2006), (Sánchez-rivera et al., 2008), (González Acevedo & Villamizar Galvis, 2019), (Berga Cabello, 2012), (Muro-cuellar & Velasco-Villa, 2004), (González Muñoz & Balbin Vallejo, 2015), (Aguirre Marrett & Larco Torres, 2010), (Reyes Sierra & Montaña Ortega, 2010), (Takaki & Mori, 2006), (Jama Barzola & Cordero Correa, 2011), (Becerra Terán & Loyola Cisneros, 2019), (Ponce López & Boada Herrera, 2006), (Botella et al., 2013), (González Rodríguez et al., 2011), (Rentería, 2009), (Li & Dai, 2006), (Yan et al., 2007), (Huang et al., 2010), (Nayyeri et al., 2011), (Li et al., 2011), (Herrera Garzón & Romero Torres, 2010), (Noda & Ramirez, 2007), (Páez Chávez & Amador Moreno, 2005)
	INSTRUMENTATION	(Hernández Calderón et al., 2011), (Zarudi & Shenkman, 1993), (Mesa Villanueva, 2011), (Alvarez Villanueva & Gonzalez Rugel, 2005), (García Fominaya, 2004), (Hernández González et al., 2014), (Nguyen & Li, 2007), (Shakibaei et al., 2014), (Montalvo Martín, 2010), (Arcos Casarrubias et al., 2003), (Bosch Neto et al., 2019), (bernal Mejía & Espinoza Siguencia, 2013), (Tommasini et al., 2008)
MEDICAL		(Bozkurt et al., 2005), (Pedersen et al., 2010), (Torres Candebat et al., 2012), (Reyes et al., 2010), (Dominguez Almaguer, 2003), (Rappaport et al., 2003), (Santra et al., 2019), (LIcona Rios & Lozano González, 2006), (Ibarra Hernández, 2014)
TELECOMMUNICATIONS		(James c., 1968), (Moroz & Borovets, 2017), (Bozkurt et al., 2015), (Fernández Martinez, 2018), (Alvizuri Romani, 2006), (López Uribe & Selene, 2008), (Francisco & Hernando, 1993), (Camarena Ibarrola, 2011), (Pérez Lance, 2019), (Martins, 2009), (Hernández Castaño et al., 2017), (Acevedo-mosqueda et al., 2010), (Hernández Castaño, 2017), (Chipule Pérez et al., 2016), (Arcos Pantoja & Calderón Chávez, 2002), (Melo Fernandez & Ruiz Toledo, 1967), (Juménez Galindo, 2018), (Jurado Moreno, 1998), (Apolo Castillo & Córdova Medina, 2010)

Then, in order to present a control engineering application and reinforce the baseline given for present and future research in control and automation, a random case of a plant-based system with a continuous-time controller is chosen and the best controller is determined by analyzing and simulating in Matlab® by applying Z-transform to treat the system in discrete time domain.

Figure 1. First segment of Z transform categorization and subcategorization. Source: Own elaboration.

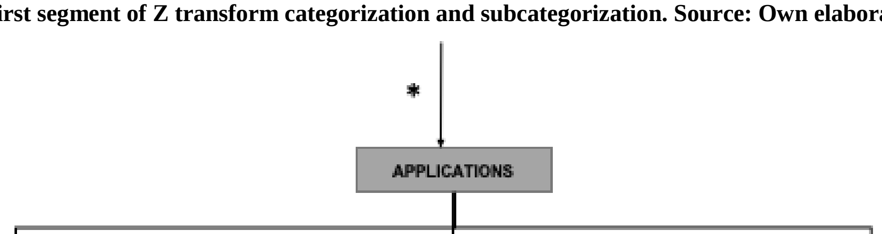
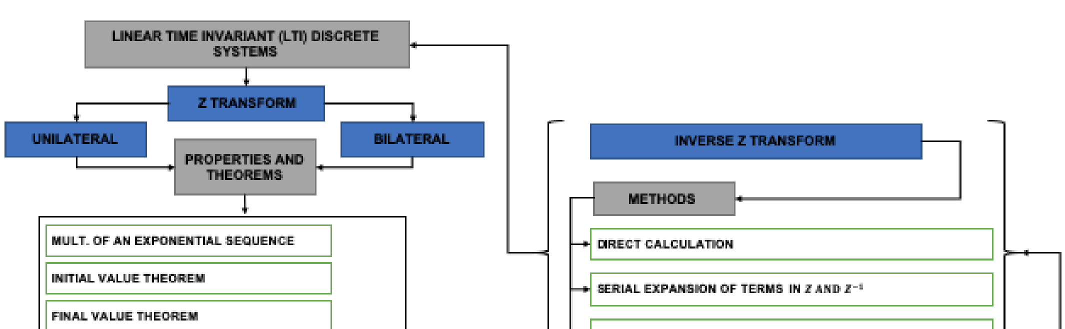


Figure 2. Second segment of Z transform categorization and subcategorization. Source: Own elaboration



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Source description and interpretation

For industrial applications with their control and instrumentation classifications, they are described and interpreted as follows:

Control

Digital controllers are used to achieve stable performance, in terms of maximum productivity, maximum profit, minimum cost or minimum energy utilization. This has led to seek to control certain aspects in a plant or process depending on the application conditions.

Initially, in systems whose main action is to control the speed using Z transform, the following applications stand out:

- Tools designed and developed in order to identify parameters of linear and nonlinear systems off-line, (Orjuela & Chica, 2007).
- Application of adaptive control, based on a reference model, on a Siemens S7 family micro-PLC, (Pérez Ramírez et al., 2019)**.**
- Controller design to have an output, position, following the reference signal in sinusoidal form, (Bory-prevez & Chang-Mumañ, 2017).
- Vector control systems using dead time controlled PWM inverters with output LC filter, (Kojima et al., 2004).
- WinCC SCADA system, where an application allows the integral proportional control of a first order RC network for the identification of a DC motor, (Sabin & González, 2009).
- High speed simulation of electric drives using Z transfrom, (Moroz & Holovach, 2017).

- Implementation of an adaptive controller based on a reference model, applied to a first order process with varying pole and gain, through a digital signal processor (DSP), (Pérez R et al., 2006).
- Implementation of control strategies using Field Programmable Gate Array (FPGA) reconfigurable devices for time-delayed linear unstable systems, (Sánchez-rivera et al., 2008).
- Implementation of Object Linking and Embedding for Process Control (OPC) communication protocol between MatLab® software and LabVIEW® graphical programming platform, to link them with two programmable logic controllers (PLC): Siemens S7-300® and Allen Bradley Control Logix 5566®, and a Festo® workstation, (González Acevedo & Villamizar Galvis, 2019).
- Hardware implementation of a digital control system for an inverted pendulum system, (Berga Cabello, 2012).

On the other hand, in systems whose main action is to control temperature using Z transform, applications stand out as follows:

- Regulation of systems with delays using an approximate discrete approach (application to recirculating systems), (Muro-cuellar & Velasco-Villa, 2004).
- The design of a universal digital PID controller with industrial-type features, (González Muñoz & Balbin Vallejo, 2015).
- Identification and design of a temperature controller for a temperature control system in a thermal room, (Aguirre Marrett & Larco Torres, 2010).
- Modeling and digital temperature control for electric furnaces, (Reyes Sierra & Montaña Ortega, 2010) .

In the case of systems whose main action is to control level using Z transform, the following applications stand out:

- Generalized minimum variation control for MIMO system with multiple time delays, (Takaki & Mori, 2006).
- Identification and design of liquid level controller for a liquid level regulating system in a tank, (Jama Barzola & Cordero Correa, 2011).
- Design and implementation of a discrete PID algorithm to increase control loops in programmable logic controllers, (Becerra Terán & Loyola Cisneros, 2019).
- Implementation of a ball and beam control system using MatLab®, (Ponce López & Boada Herrera, 2006).

And, from another approach, computer control applications have made "intelligent" motion possible in robotic systems; some of the applications that employ Z transform include:

- Modeling bioinspired temporal filters for motion estimation, (Botella et al., 2013).
- Control for manipulator robots with visual feedback and electro-pneumatic platform 3gdl, (González Rodríguez et al., 2011).

Regarding safety system controls using Z transform, one application stands out:

- The study of ultrasonic systems and their application to the active safety in intelligent vehicles, (Rentería, 2009). In this research, the possibility of using an ultrasonic system as a sensor element for intelligent vehicle control in urban traffic was analyzed, using Z transform in order to model some vehicle components and to study digital filters used for processing ultrasonic signals.

In magnetic system controls using Z transform, the following applications are described:

- Z transform implementations in CFS-PMLs, (Li & Dai, 2006); as well as the FDTD analysis - based on Z-transform - of a perfectly conducting cylinder covered with non-magnetized plasma, (Yan et al., 2007).
- Improved $(FD)^2$ TD formulations - based on the Z-transform - for anisotropic plasma,, (Huang et al., 2010).
- FDTD modeling of dispersive bianisotropic media using *Z* transform method , (Nayyeri et al., 2011).
- Efficient implementation of higher-order LMP based on *Z* transform method, (Li et al., 2011).
- Digital controller for a magnetic levitation plant with one degree of freedom, (Herrera Garzón & Romero Torres, 2010).
- Z transform based methods for electromagnetic transient simulations, (Noda & Ramirez, 2007).
- Implementation of a magnetic levitation system controlled by means of MatLab®, (Páez Chávez & Amador Moreno, 2005).

Instrumentation

The growing trend to control dynamic systems digitally instead of analogically, is mainly due to the availability of low cost digital computers and the advantages of working with digital signals instead of continuous time signals; but all of this is also due to the help of equipment used to obtain data or devices used to perform this tasks. The following is a list of the sources that, using Z transform, stand out for this type of subcategory:

- Discrete model of a DC generator by Z transform and difference equations, (Hernández Calderón et al., 2011).
- Analysis of common switching converters using Z transform, (Zarudi & Shenkman, 1993).
- Measurement of acoustic parameters in enclosures and spectral compensation system, (Mesa Villanueva, 2011).
- Design and implementation of a frequency and voltage control system using MatLab ® , (Alvarez Villanueva & Gonzalez Rugel, 2005).
- Classification and identification for surface and high-resolution surveillance radars in lpi scenarios, (García Fominaya, 2004).
- Analysis and design of a control scheme for portable dry bath application, (Hernández González et al., 2014).
- Application of a Z transform signal model and median filtering for phasor and frequency measurements of an electrical system. (Nguyen & Li, 2007).
- Implementation of half-dispersive IIR filter using *Z* transform, (Shakibaei et al., 2014).
- Signal analysis of mechanical measurements for advanced predictive maintenance, (Montalvo Martín, 2010).
- Reaction kinetics and its influence on the distribution of molecular weights in an emulsion-seeded polymerization system, (Arcos Casarrubias et al., 2003).
- Application of *Z* transform in biopolymers production using sugarcane bagasse, (Bosch Neto et al., 2019).
- Implementation of a 3-component accelerograph, based on a kinemetrics fba-23, with geo-referencing and communication system, (Bernal Mejía & Espinoza Siguencia, 2013).
- Efficient modeling and objective validation of an indoor impulsive response, (Tommasini et al., 2008).

Case study of a practical application of Z transform in the field of control engineering.
Guerrero Naboa, in 2012, published an investigation to determine PID controller parameters for a plant with a known transfer function; then, the controller is designed based on the plant transfer function that generates a better response of the control system. Initially, a unitary step reference input is considered; a maximum over-elongation of 20% and a minimum of 2% must be complied with, and a settling time less than 4 seconds is sought. The plant transfer function is given in (22):

$$\frac{C(s)}{R(s)} = \frac{1}{s(s+1)(s+5)} \tag{22}$$

The above is proposed in continuous time systems; now it will be developed in discrete time applying the Z transform; as well as several MatLab® R2020a tools that ease case analysis.

- The process begins by taking the whole system (transfer function) to MatLab®.
- This review detects a procedural error in the original document, since the transfer function, when transcribed in MatLab®, is raised one more degree; therefore, all system conditions are changed. It is enough to show that all the following process - where controller calculations are carried out in continuous time - lead to a completely different system from the original problem.

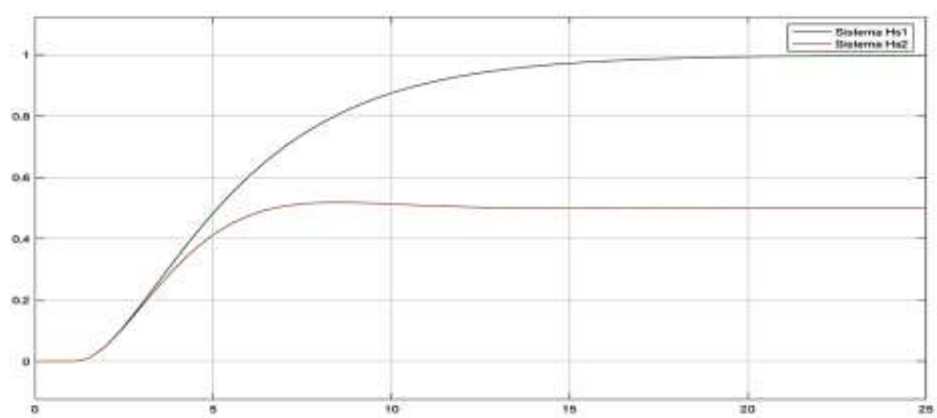
I. Problem system: $$Hs1 = \frac{1}{s(s+1)(s+5)} = \frac{1}{s^3+6s^2+5s} \tag{23}$$

II. System used by Guerrero Naboa:

$$Hs2 = \frac{1}{s^3+6s^2+5s+1} \tag{24}$$

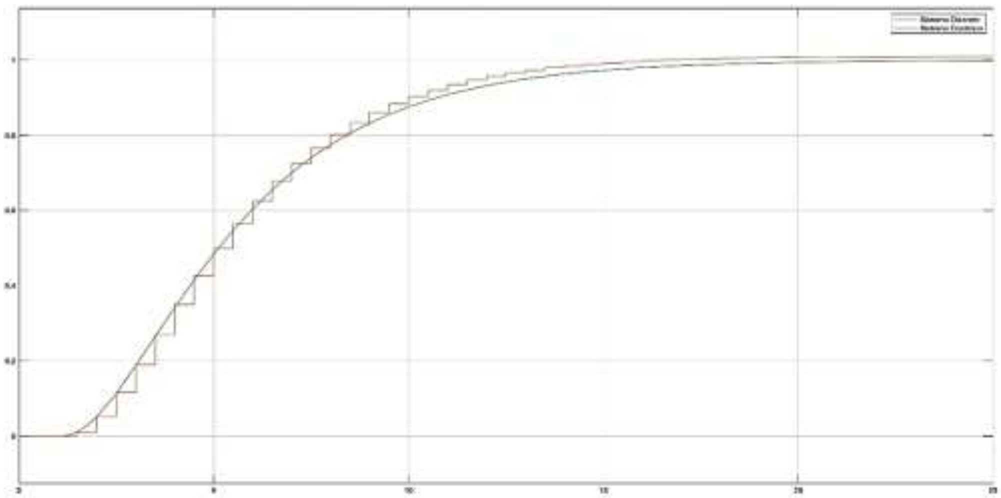
- When plotting the systems against a closed-loop step input, the difference is evident, Fig. 1.

Figure 1. Hs1 VS Hs2 System at unitary step input. Source: Own elaboration



- In order to work the original system according to the parameters required by reference terms, the transfer function is transformed from continuous time to discrete time with Ts= 0.5s in order to bring the two functions closer to each other.
- Now, simulating both functions at a closed-loop step input, Fig. 2 is obtainded.

Figure 2.



Continuous VS Discrete at unitary step input. Source: Own elaboration

With the system behavior models for a closed-loop step input, the discrete-time transfer function is taken to MatLab® SISOTOOL to design the controller, as follows:

- Architecture selection to be used, in this case configuration 1.
- Transfer function selection.
- The transfer function is plotted in order to determine system behavior parameters:

  Rising time: 7,68 s

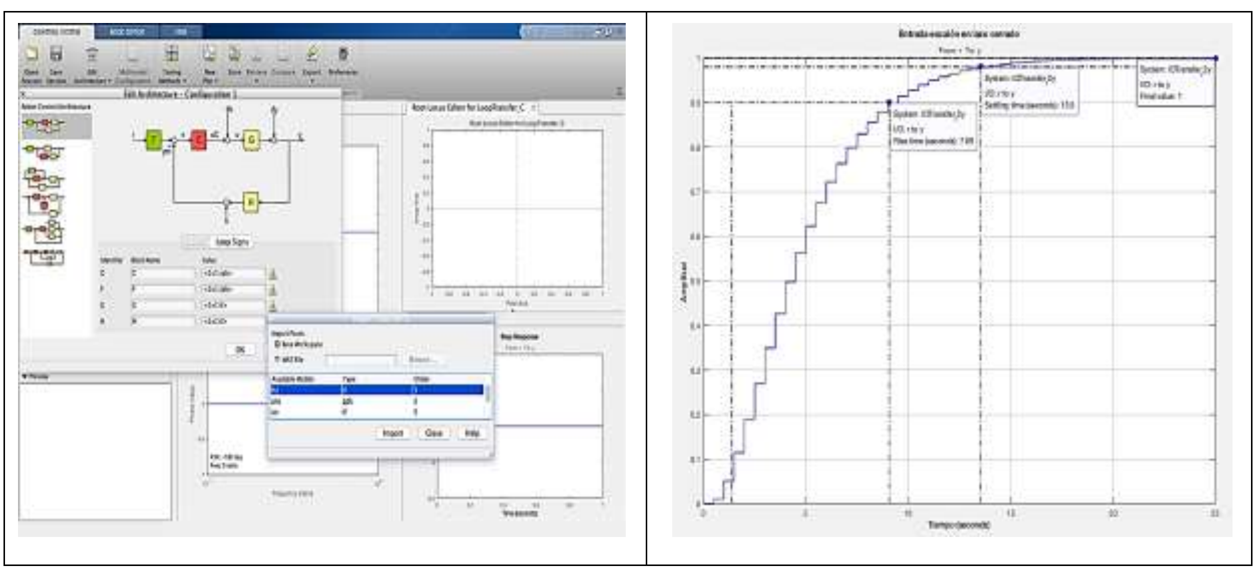
  Settling time (2% channel): 13,5 s

  Over elongation: No.

Fig. 3 is obtained.

Now, it is necessary to select the best controller to meet system requirements. In the root location window are set system constraint parameters: maximum over-elongation of 20% and minimum of 2%, and also a settling time of less than 4 seconds, this will indicate the ideal design region and opens the PID Tuning application, see Fig.4.

Figure 3. Transfer function selection and discrete time system data of the original function response to step input in closed loop. Source: Own elaboration.



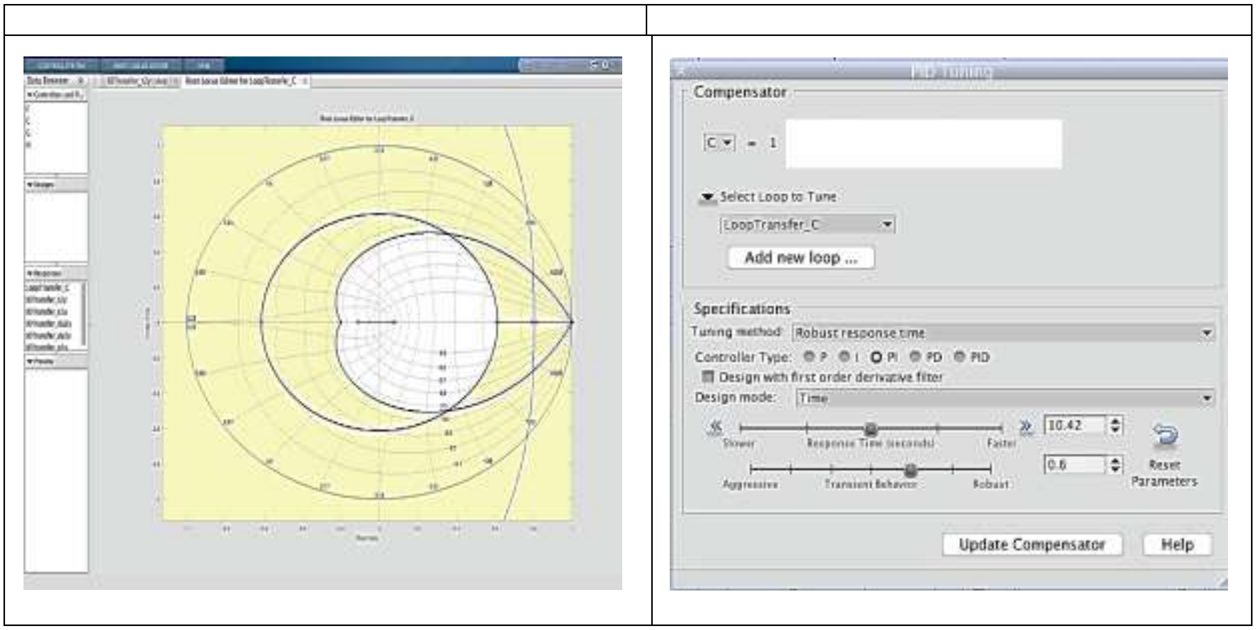
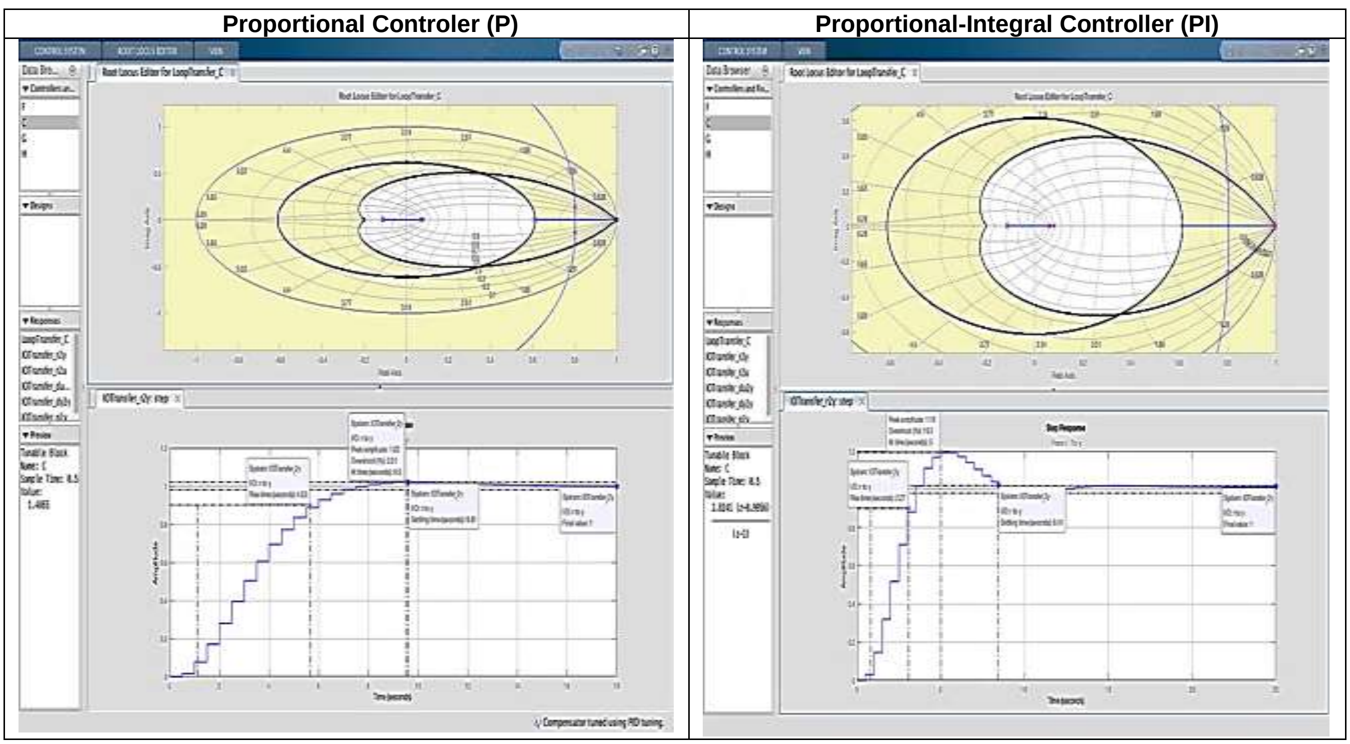


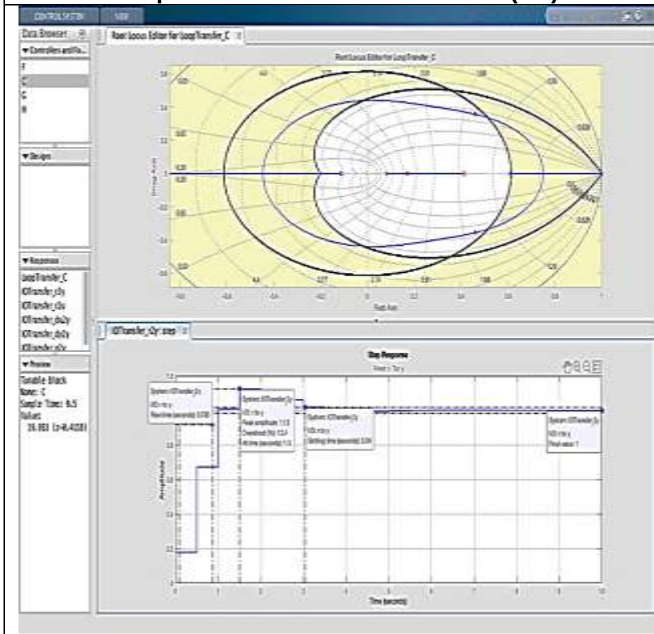
Figure 4. Parameter restriction selection and PID tuning tool. Source: Own elaboration.

Then with PID tuning, the best system controller begins to be selected. Figures 5 and 6.

Figure 5. System response with P and PI controllers. Source: Own elaboration.



Proportional-Derivative Controller (PD)	Proportional-Integral-Derivative Controller (PID)



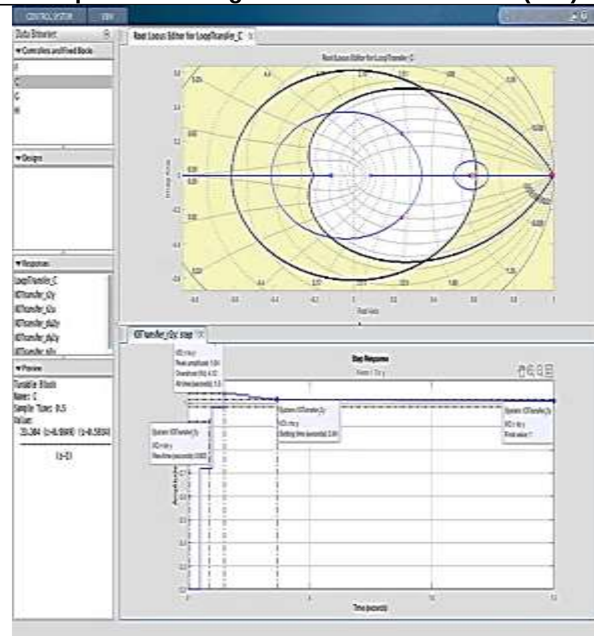


Figure 6. System response with PD and PID controllers. Source: Own elaboration

The following is a comparative analysis of the responses of each of the configured controllers, see table 4.

Controller	Original System	P	PI	PD	PID
Rising time (s)	7,69	4,53	2,27	0,783	0,805
Settling time (s)	13,5	9,61	8,41	3,04	3,64
Over-impulse (%)	N.A.	2,01	19,2	12,4	4,12

Table 4. Controller response comparison. Source: Own elaboration

By comparing each of the responses of every controller that was developed it can be assured that two of them are within the requirements of the problem. Although the PD controller has a better settling time it outperforms the PID controller over elongation by 3 times; therefore, PID is the best controller to employ in this kind of discrete system.

Conclusions

- It has been found that 30% of the total of the references produced by the review is made up of speed control systems, temperature, level and magnetic type systems. From the above it can be concluded that these control systems are predominant in the industries due to the fact that they are the most common structures to develop and where it is one of the most likely areas of application of Z transform.
- Although there are several identification methods –both parametric and non-parametric - that provide a variety of information about the dynamics of a plant, it is plausible to develop or design controllers according to the process. Taking into account the above, all the applications

previously discussed in section 4.1 –Control- employ methods that facilitate controller design: pole relocation, PID tuning, among others. Consequently, the results have presented numerical tests, schematics and a reasonable selection of parameters indicating that the controls are very accurate and efficient.

- Section 4.1 –Control- compares numerical results with analytical results of each one of the applications and shows a good agreement with the expected results in each project; at the same time, they have been implemented in commercial devices that are in common domain of most students, academics and engineers, resulting in solid baselines for future application development.
- From section 4.2 -Instrumentation-, the applications stand out for being the basis for measurement system instruments, processing systems, generators, converters, radars, etc. The versatility of these projects stands out for the ability to use in their development, from microcontrollers, development cards such as FPGA's or robust systems such as PLC's, to data acquisition SW's, among these: Labview® and MatLab®, which through digital or real simulations allow to demonstrate the controller efficiency and, consequently the usefulness of the approximate models in the developed discrete systems.
- Although robotic control systems are found in almost all industries and automated systems in this categorization only cover 4% of the total reviewed references, due to the fact that the sources are mostly theses, articles or academic texts. Because of the complexity of robot development, these applications correspond to studies of large companies and industrial developers from which obtaining information is limited; i.e., it can be a great area of research and application of the *Z* transform.
- According to this research, an adequate state of knowledge is obtained in order to serve as a baseline for research on potential categorizations of Z transform, specifically. It is evident that the developed categorization is open to be complemented with many more categories and subcategories because Z transform is found in digital systems, and this study was specifically emphasized in control and instrumentation systems. For other applications susceptible to use similar methods described in this review, information intensive scenarios and different application contexts can be suggested: (Nieto Duran, et al., 2017; Duarte Barón & Borrás Pinilla, 2016; Caicedo, & Smida, 2016; Montiel, et al., 2016; Briñez de León, et al., 2017; Herrera-Cubides, et al., 2019; Salamanca, 2018; Angulo, et al, 2017; Lemus, et al., 2016; López, et al., 2018).

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